DETERMINATION OF WELL INFLUENCE COEFFICIENTS ON THE BASIS OF PERTURBATIONS OF THE STEADY STATE

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In a porous bed, in which the percolation is determined by Darcy's equation, the well bottom pressures and flow rates are given by the linear relations

$$p_0 - p_i = \sum_{j=1}^{n} a_{ij} q_j \qquad (i = 1, 2, \ldots, n). \tag{1}$$

Here, p_0 is the pressure on the pool supply contour, p_i and q_i are the bottom pressure and flow rate of the i-th well, n is the number of wells in the area, and a_{ij} are influence coefficients. Knowing these, we can determine the total extraction of liquid, the distribution of well flow rates, the degree of extraction of oil, the yield of deposit water, the optimum flooding percentage, etc.

The influence coefficients depend on the pool geometry, the location of the wells, and the physical parameters of the bed and the saturating liquid. At present they are determined on an electrical model of the bed, whose construction is a rather laborious affair, especially in the case of three-dimensional beds. Moreover, this method does not always yield values of the influence coefficients with the necessary accuracy, since the model does not completely reflect the complex local conditions. Consequently, direct methods of determining these coefficients directly in the field are of practical importance. One such method is proposed below.

Considering Eq. (1), we see that if a certain well, for example, the j-th, is "instantaneously" shut down, after a sufficiently long interval of time the pressure in the i-th well will increase by an amount $\Delta p_i = a_{ij} q_j$, the case i = j not being excluded. However, in the transition period, when a steady state has not yet been established, the latter quantity will be a variable. Let it vary according to the law

$$\delta p_i(t) = a_{ij}(t) q_j, \tag{2}$$

in which $a_{ij}(t)$ is an unknown but perfectly definite function depending on the same factors as the influence coefficient a_{ij}. Comparing Eq. (2) with the previous equation, in the limit we have

$$a_{ij}(\infty) = a_{ij}, \quad \delta p_i(\infty) = \Delta p_i.$$

Now let $\delta q_j(t)$ be some variation of the flow rate in the j-th well, and $\delta p_i(t)$ the corresponding change in pressure in the i-th well; then the relation between them is established via the function $a_{ij}(t)$ introduced in (2) by means of the Duhamel formula

$$\delta p_i(t) = \frac{d}{dt}\int_0^t a_{ij}(t-\sigma)\,\delta q_j(\sigma)\,d\sigma. \tag{3}$$

Obviously, when $\delta q_j = q_j$ = const Eqs. (2) and (3) coincide.

We will describe as a perturbation of the stationary state of a well (state of rest or steady operation) a change in the pressure and flow rate for which the latter, after a certain interval whose duration is not limited, assume their original values. For idle wells such a perturbation may be produced by briefly starting up and shutting down operation, for operating wells by a brief shutdown, after which the well is started up again in order to restore completely the original operating conditions, etc.

Let $\delta p_i(t)$ and $\delta q_j(t)$ denote the deviations of the bottom pressure and flow rate from the original values in the presence of such a perturbation. Then by definition the integrals

$$\int_0^\infty \delta p_i(t)\,dt, \qquad \int_0^\infty \delta q_j(t)\,dt$$

have a finite value.

Subjecting (3) to a Laplace transformation, we obtain

$$s a_{ij}^*(s) = \frac{\delta p_i^*(s)}{\delta q_j^*(s)} \tag{4}$$

$$\left(\delta p_i^*(s) = \int_0^\infty e^{-st}\,\delta p_i(t)\,dt, \ldots\right).$$

Using the limit relations of operational calculus [1], we have

$$\lim_{s\to 0} s a_{ij}^*(s) = a_{ij}(\infty) = a_{ij},$$

$$\lim_{s\to 0} \delta p_i^*(s) = \int_0^\infty \delta p_i(t)\,dt, \quad \lim_{s\to 0} \delta q_i^*(s) = \int_0^\infty \delta q_j(t)\,dt.$$

Then passing to the limit as s → 0 in Eq. (4) and using the above equations, we obtain the following expression for the influence coefficients:

$$a_{ij} = \left(\int_0^\infty \delta p_i(t)\,dt\right)\left(\int_0^\infty \delta q_j(t)\,dt\right)^{-1}. \tag{5}$$

This equation opens up the possibility of determining the influence coefficients on the basis of a direct hydrodynamic investigation of the well.

Equation (5) is suitable for determining the influence coefficients not only of porous beds but also of porous-fissured beds in the sense of [2], since the basic premise—the validity of Darcy's law—still applies. However, unless corrections are introduced, Eq. (5) is not suitable for beds characterized by a nonlinear percolation regime.

We now present two examples of the practical application of Eq. (5).

Example 1. Under steady-state conditions a well produces q_i = = 206 m^2/day = 2400 cm^3/sec. In order to obtain $\delta p_i(t)$ and $\delta q_i(t)$ it is shut down for 70 min, then started up again. The corresponding dynamics of the variation in pressure and flow rate are shown in Fig. 1, from which it is clear that $\delta p_i(t) = 0$ after only t = 110 min. The flow rate, however, is much slower in recovering and reaches the original value after four hours. From Fig. 1 we find

$$\int_0^\infty \delta p_i(t)\,dt = 16.59\cdot 10^5 \text{ at sec}, \quad \int_0^\infty \delta q_i(t)\,dt = 69\cdot 10^6\ \text{cm}^3.$$

The self-influence coefficient a_{ii} = 0.024044 at sec/cm^3.

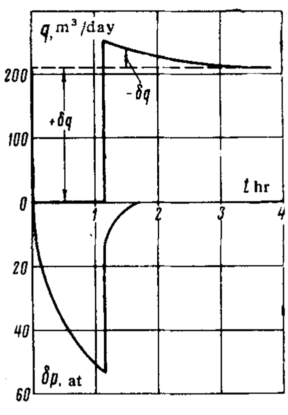

Fig. 1

In this specific case a knowledge of a_{ii} enables us to find the steady-state depression, i.e., the deviation of the steady-state value of the bottom pressure from the nominal static value, from the formula $\Delta p_i = a_{ii}q_i$.

Using this formula, we find $\Delta p_i = 0.024044 \cdot 2400 = 57.7$ at.

Direct measurements during an extended shutdown of the well (5 hr) on the next day showed that $\Delta p_i = 59$ at. The error is clearly small.

This example shows how it is possible to determine the steady-state and hence the formational (nominal static) pressure during a brief shutdown of the well, without serious pumping losses.

Example 2. A test well was started up and shut down after 9 hr. The dynamics of the pressure and flow variation corresponding to this disturbance of the stationary state are shown in Fig. 2, from which it is clear that the pressure variation almost ceased after 40 hr. From Fig. 2 we find

$$\int_0^\infty \delta p_i(t)\, dt = 1.205 \cdot 10^6 \text{ at sec},$$

$$\int_0^\infty \delta q_i(t)\, dt = 21 \cdot 10^6 \text{ cm}^3 .$$

Consequently, for this well

$$a_{ii} = 0.0574 \text{ at sec/cm}^3.$$

In this case, using a_{ii}, we can establish the productive potential of the well, i.e., its output after a sufficiently long period of operation. This is especially important for test wells in areas without facilities for storing the oil, where the productive potential cannot be established by operating them over an extended period. In these cases it is usual to employ the formula

$$q_i = \frac{2\pi kh}{\mu} \frac{\Delta p_i}{\ln (r_1/r_2)} .$$

Here, q_i is the potential output of the well; Δp_i is the depression at which it will be operated; kh/μ is the hydraulic conductivity of the bed; r_1/r_2 are the radii of the supply contour and the well, respectively. The parameter kh/μ is determined by a suitable express method; the ratio of the radii is found indirectly and rather arbitrarily. With this

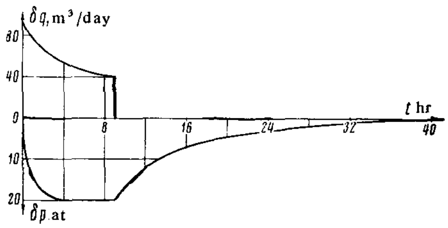

Fig. 2

approach the Dupuit formula may give only a very rough idea of the relative productive potential of the well. When the influence coefficient is used these shortcomings can be avoided. In fact, the productive potential can be determined from the formula

$$q_i = \Delta p_i / a_{ii} .$$

In particular, for the test well in question

$$q_i = 348 \text{ cm/sec} = 30 \text{ m}^3\text{/day} \quad \text{at} \quad \Delta p_i = 20 \text{ at}.$$

Owing to the lack of test data it is not possible to present examples of the determination of mutual influence coefficients (in the strict sense of the word).

Clearly, the proposed method of determining the influence coefficients is reasonably simple and can be recommended for practical use.

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